N_α-Continuous And Contra-N_α-Continuous Mappings

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الخلاصة

في هذا البحث قدمنا أنواع جديدة من التطبيقات المستمرة من النمط Nα باستخدام المجموعات المفتوحة من النمط Nα في الفضاءات التبولوجية مثل التطبيقات المستمرة من نمط Nα ، $N_{\acute{a}}^{*}$ ، $N_{\acute{a}}^{**}$ وكذلك درسنا بعض خصائص هذه الأنواع علاوة على ذلك درسنا بعض أصناف التطبيقات العكسية المستمرة التي تسمى التطبيقات العكسية المستمرة من نمط Nα وبينا العلاقات بين هذه الأنواع.

الكلمات المفتاحية

المجموعة المفتوحة -α ، المجموعة المفتوحة -N_α، الفضاءات التوبولوجية N_α.

Abstract

In this paper, we introduce new types of N_α-continuous mappings by using N_α-open sets in topological spaces, such as N_α-($N_{\acute{a}}^{*}$, $N_{\acute{a}}^{**}$) continuous mappings , also we study some properties of these types. Moreover, we study some classes of contra-continuous mappings called contra N_α-continuous and show relationships between these types.

Keywords

α-open set, N_α-open set, N_α-regular space.

1. Introduction

The concept of N_α-open set was first studied in 2015 by N. A. Dawood, N. M. Ali ,see [1] by using these sets we study some class of continuity mappings which are N_α-($N_{\acute{a}}^{*}$, $N_{\acute{a}}^{**}$) continuous mappings and investigated some of their properties. The notion of contra-continuity was first investigated by Dontchev in 1996, [2]. Subsequently, Jafari and Noiri [3,4] exhibited contra-α-continuous, and contra-pre-continuous mapping. A good number of researchers have also initiated different types of contra continuous mappings, some of which are found in the papers [5-9]. Here, in this paper also, attempt has been made to employ the notion of N_α-open sets to study some variation of contra continuous mappings called contra-N_α-continuous mappings.

In this paper all spaces X and Y are topological spaces, also the closure (interior resp.) of a subset A of X is denoted by cl(A) (int(A) resp).

2. Some Basic Concepts

Here, we shall give some basic concepts which we need in our work.

2.1. Definition [10]

Let (X,τ) be a topological space, a subset A of X is called α-open if $A \subseteq$ int cl int (A). The complement is called α-closed.

From the above definition it is easy to check that, every open is α-open, [11].

2.2. Definition [12], [13]

Let (X,τ) be a topological space, a subset A of X is called :

(1) regular-open if A = int cl(A)

(2) θ-open if for each $x \in A$, there exists open set B such that $x \in B \subseteq$ cl $B \subseteq A$.

2.3. Definition [14], [15], [16], [6]

A mapping $f : X \longrightarrow Y$ is called α-continuous (perfectly continuous, stronglyθ - continuous, regular closed continuous), if every an open set A in Y, then $f^{-1}(A)$ is α-open (clopen, θ-open, regular closed resp.)in X.

2.4. Definition [1]

Let (X,τ) be a topological space, a subset A of X is called "N_α-open" set if there exists a non-empty α-open set B such that cl $B \subseteq A$.

The family of all N_α-open sets is denoted by $N_\alpha O(X)$, and its complement is called N_α-closed and denoted by $N_\alpha C(X)$.

2.5. Remark [1]

In every topological space the set X is N_α-open set.

2.6. Remarks [1]

(1) The concepts of open and N_α-open sets are independent.

(2) The concepts of α-open and N_α-open sets are independent.

(3) The concepts of closed and N_α-open sets are independent.

(4) Every clopen set is N_α-open set.

(5) Every θ-open set is N_α-open set.

(6) Every closed α-open set is N_α-open set.

2.7.Theorem [1]

Let (X_1,τ_1), (X_2,τ_2) be topological spaces. Then A_1 and A_2 are N_α-open(N_α-closed) sets in X_1 and X_2 resp. if and only if $A_1 \times A_2$ is N_α-open(N_α-closed) set in $X_1 \times X_2$.

2.8. Proposition [1]

Let (X,τ) be a topological space. Then

(1) The finite union of N_α-open sets is N_α-open set.

(2) The finite intersection of N_α-open sets is N_α-open set.

(3) The finite union of N_α-closed sets is N_α-closed set.

(4) The finite intersection of N_α-closed sets is N_α-closed set.

2.9. Definition [1]

Let (X,τ) be a topological space ,$A\subseteq X$. The N_α-closure of A is defined as the intersection of all N_α-closed sets in X containing A, and is denoted by $N_\alpha\, cl(A)$.

2.10. Lemma [1]

If (X,τ) is a topological space , where $A \subseteq B \subseteq X$, then

(1) $N_\alpha\, cl(A) \subseteq N_\alpha\, cl(B)$.

(2) If A is N_α-closed set, then $A = N_\alpha\, cl(A)$.

(3) $x \in N_\alpha\, cl(A)$ if and only if $U_x \cap A \neq \phi$ for any N_α-open set U containing x.

2.11. Proposition [1]

Let (Y,τ_Y) be a subspace of a topological (X,τ) such that $A \subseteq Y \subseteq X$. Then

(1) If $A \in N_\alpha O(X)$, then $A \in N_\alpha O(Y)$.

(2) If $A \in N_\alpha\,(Y)$ then $A \in N_\alpha(X)$,where Y is clopen set in X .

2.12. Definition [11]

Let (X,τ) be a topological space .Then X is called α**-regular space if for every $x\in X$,and every α-closed set F such that $x\notin F$ there exist two open sets A and B such that $x\in A$, $F\subset B$ and $A\cap B=\varnothing$

2.13. Definition [1]

Let (X,τ) be a topological space. Then X is called $N_{\acute{a}}^{**}$ -regular space if for every

$x\in X$,and every Nα- closed set F such $x\notin F$ there exist two open sets A and B such that $x\in A$, $F\subset B$ and $A\cap B=\varnothing$

2.14. Proposition [11] , [1]

Let (X,τ) be a topological space . Then :

(1) X is α**-regular space iff every an α-open set A contains x, there exists an open set B contains x such that $x \in B \subseteq cl\, B \subseteq A$.

(2) X is $N_{\acute{a}}^{**}$ -regular space if and only if every N_α-open set A contains x, there exists an open set B contains x such that $x \in B \subseteq cl\, B \subseteq A$.

2.15. Proposition [1]

Let (X,τ) be α**-regular space. Then

(i) Any an α-open set ($_\alpha$-closed) is N_α-open set (N_α-closed).

(ii) Any an open set(closed) is N_α-open set (N_α-closed).

2.16. Proposition [1]

Let (X,τ) be $N_{\acute{a}}^{**}$ -regular space .Then

(i) Any N_α-open (N_α-closed) set is an open(closed) set.

(ii) Any N_α-open (N_α-closed) set is an α-open($_\alpha$-closed) set.

2.17. Definition [17]

Let (X,τ) be a topological space. Then X is called Ultra-T2 space if for each pair of distinct points x and y ,there exist clopen sets A and B containing x and y resp. such that $A\cap B=\varnothing$

2.18. Definition [18]

Let (X,τ) be a topological space. Then X is called locally indiscrete if every open set of X is closed.

3. Some Types of N_α-Continuity

In this section, the concept of N_α-open set will be used to define some new types of N_α-continuity such as; N_α-continuous, $N_{á}^{*}$-continuous and $N_{á}^{**}$-continuous. Moreover we shall study the relationships with other some types of continuity mappings.

3.1. Definition

Let (X_1,τ_1), (X_2,τ_2) be topological spaces, such that $f: X_1 \longrightarrow X_2$ any mapping. Then f is N_α-continuous if for each an open set A in X_2, then $f^{-1}(A)$ is N_α-open set in X_1.

3.2. Remark

There is no relation between the continuous and N_α-continuous mappings ,we shall explain this in Example (3.3).

3.3. Example

Let (X,τ_1) be a topological space, where $X=\{1,2,3,4\}$, $\tau_1=\{X,\{2\},\{1,4\},\{1,2,4\},\phi\}$, $\tau_2=\{X,\{1\},\{1,2,3\},\phi\}$ and $f:(X,\tau_1)\longrightarrow(X,\tau_2)$ is a mapping such that $f(1)=f(2)=f(4)=1$, $f(3)=3$.

Thus f is continuous which is not N_α-continuous ,since A=,{1} is an open set ,but $f^{-1}(A)=\{1,2,4\}$ which is not is N_α-open set

3.4. Remark

There is no relation between the α-continuous and N_α-continuous mapping. See previous example (3.3) where f is α-continuous which is not N_α-continuous.

Now the following Example explains the N_α-continuous mapping neither continuous nor α-continuous mapping in general.

3.5. Example

Let (X,τ_1),(X,τ_2) be topological spaces ,where $X=\{1,2,3,4\}$, $\tau_1=\{,\phi\{3\},\{1,4\},\{1,3,4\},X\}$, $\tau_2=\{\phi,\{1\},X\}$.Define $f:(X,\tau_1)\longrightarrow(X,\tau_2)$ such that $f(1)=f(2)=f(4)=1$, $f(3)=3$.

See the following Diagram

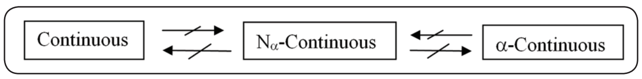

Diagram (1)

We have previously shown that there is no relationship among the concepts of continuous, α-continuous and N_α-continuous. But if we impose some conditions, then we obtain the following Diagram.

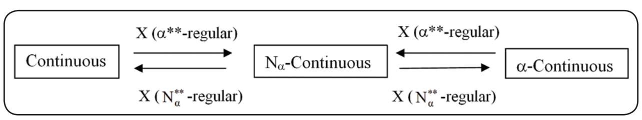

Diagram (2)

The following remark explains the relation of the concept of N_α-continuous with other types of continuity mapping such as: perfectly continuous, θ-continuous, and regular closed continuous.

3.6. Proposition

The perfectly continuous (θ-continuous-regular closed continuous resp.) is N_α-continuous.

Proof; Follows by Remarks (2.6) , Definition (2.1).

3.7. Remark

In proposition (3.6), we observe that its converse need not be true in general. See the following examples:

3.8. Examples

(1) Let (X,τ_1) , (X,τ_2) be topological spaces, where $X=\{1,2,3,4\}$, $\tau_1=\{X,\{3\},\{1,4\},\{1,3,4\},\phi\}$, $\tau_2=\{X,\{1\},\phi\}$, and $f : X \longrightarrow X$ such that f (1) = f (2) = f (4) = 1 , f (3) = 3. Thus f is N_α-continuous but it is neither perfectly continuous nor θ-continuous ,since A= {1} is an open set but $f^{-1}(A)=\{1,2,4\}$ is neither clopen set norθ-open set.

(2) Let (X,τ_1) , (X,τ_2) be topological spaces, where, $X_1=\{1,2,3,4,5\}$, $X_2=\{1,2,3,4\}$ $\tau_1=\{X_1,\{1\},\{2,3\},\{1,2,3\},\phi\}$,$\tau_2=\{X_2,\{2\},\phi\}$.

Define $f : X_1 \longrightarrow X_2$ such that f (1) = f (2) = f (4) = f (5) = 2 and f (3) = 4. Thus f is N_α-continuous which is not regular closed-continuous mapping ,since A={2}is an open set but $f^{-1}(A)=\{1,2,4,5\}$ which is not regular-closed set.

Now we have the following Diagram:

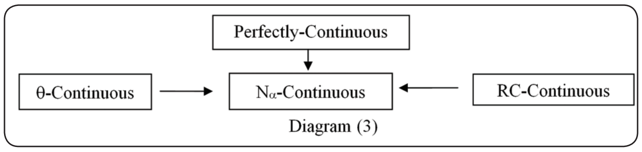

Diagram (3)

Now, we shall define other types of N_α-continuity mappings such as:

3.9. Definition

Let (X_1,τ_1), (X_2,τ_2) be topological spaces, and $f : X_1 \longrightarrow X_2$ be a mapping, then f is called

(1) $N^{*}_{\acute{a}}$ -continuous if $f^{-1}(A)$ is N_α-open set in X_1 for every N_α-open set A in X_2.

(2) $N^{**}_{\acute{a}}$ -continuous if $f^{-1}(A)$ is open set in X_1 ,for every N_α-open set in X_2

The concepts of $N^{*}_{\acute{a}}$ -continuous and $N^{**}_{\acute{a}}$ -continuous are independent .We have the following diagram.

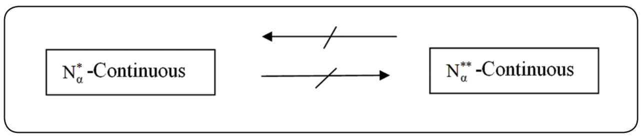

Diagram (4)

3.10. Proposition

Let (X_1,τ_1) , (X_2,τ_2) be topological spaces, and F be a subset of X_1. Let $f : X_1 \longrightarrow X_2$ be a mapping , then:

(1) If the mapping $f : X_1 \longrightarrow X_2$ is N_α($N^{*}_{\acute{a}}$ -continuous resp.), then $f_{/F} : F \longrightarrow X_2$ is also, N_α($N^{*}_{\acute{a}}$ -continuous resp.), where F is N_α-open set in X_1

(2) If the mapping $f : X_1 \longrightarrow X_2$ is $N^{**}_{\acute{a}}$ continuous, then $f_{/F} : F \longrightarrow X_2$ is also, $N^{**}_{\acute{a}}$ - continuous, where F is an open set in X_1 .

Proof: We shall prove only when the mapping f is N_α-continuous, and the other cases by the same way .Suppose B_2 is an open set in X_2, since f is N_α-

continuous, then, $f^{-1}(B_2)$ is N_α-open in X_1, also we have $f^{-1}(B_2) \cap F$ is N_α-open set in X_1(see(2.8(2)) , so it is N_α-open set in F(see proposition$_{(2.11)(1)}$). But, $(f/F(B_2))^{-1} = f^{-1}(B_2) \cap F$,thus the proof is complete.

3.11. Proposition

Let $(X_1,\tau_1)(X_2,\tau_2)$ be two topological spaces, and $f:(X_1, \tau_1) \longrightarrow (X_2,\tau_2)$ be a mapping ,where A_1 and A_2 be subsets in X_1, such that $X_1 = A_1 \cup A_2$,then:

(1) f is N_α ($N^*_{\acute{a}}$ -continuous), such that) $f|_{A1}$, $f|_{A2}$ are N_α ($N^*_{\acute{a}}$ -continuous) mappings ,where A_1 and A_2 are disjoint clopen subsets in X_1.

(2) f is $N^{**}_{\acute{a}}$ -continuous such that) $f|_{A1}$, $f|_{A2}$ are $N^{**}_{\acute{a}}$ -continuous mappings ,where A_1 and A_2 are disjoint open subsets in X_1.

proof : we shall prove only the state of N_α continuous. Suppose B is an open set in X_2, thus, $f^{-1}(B) = (f|_{A1})^{-1}(B) \cup (f|_{A2})^{-1}(B)$, but $f|_{A1}$, $f|_{A2}$ are N_α-continuous this implies, $(f|_{A1})^{-1}(B)$, $(f|_{A2})^{-1}(B)$ are N_α-open subsets in A_1, A_2 resp., since A_1 and A_2 are clopen sets in X_1 then by (proposition (2.11(2)we get, $(f|_{A1})^{-1}(B)$, $(f|_{A2})^{-1}(B)$ are N_α-open sets in X_1 ,also $(f|_A)^{-1}(B) \cup (f|_{A2})^{-1}(B)$ is N_α-open set in X_1 this, implies $f^{-1}(B)$ is N_α-open set in X_1.

3.12. Proposition

Let (X_1,τ_1) , (X_2,τ_2) be topological spaces, let $f: X_1 \longrightarrow X_2$,and $f_A: f^{-1}(A) \longrightarrow A$ which defined by , $f_A(x)=f(x)$ be mappings . We have the fowlloing:

(1) If f is N_α-continuous ,then f_A is also, N_α-continuous ,where A is an open set in X_2

(2) If f is $N^*_{\acute{a}}$ ($N^{**}_{\acute{a}}$ -continuous),then f_A is also, $N^*_{\acute{a}}$ ($N^{**}_{\acute{a}}$ continuous),where A is clopen set in X_2.

Proof: We choose(1) (2) ,and the other case is similarly. Suppose B is open set in A, since A is open in X_2,then B is open in X_2,since f is N_α-continuous thus $f^{-1}(B)$ is N_α-open set in X_1,since $f^{-1}(B) \subseteq f^{-1}(A) \subseteq X_1$,then by (proposition(2.11(1)), we get $f^{-1}(B)$ is N_α-open set in $f^{-1}(A)$.

The proof of (2) by using proposition(2.11(2)).

3.13. Proposition

Let $(X_1,\tau_1)(X_2,\tau_2)$ and (X_3,τ_3) be topological spaces and $f:(X_1,\tau_1) \longrightarrow (X_2,\tau_2)$ be a mapping then :

(i) If $f: X_1 \longrightarrow X_2$ is N_α-continuous and $X_2 \subseteq X_3$,then $f: X_1 \longrightarrow X_3$ is also N_α-continuous.

(ii) If $f: X_1 \longrightarrow X_2$ is $N^*_{\acute{a}}$ ($N^{**}_{\acute{a}}$ -continuous),and $X_2 \subseteq X_3$, then $f: X_1 \longrightarrow X_3$ is also $N^*_{\acute{a}}$ ($N^{**}_{\acute{a}}$ -continuous).

Proof : we shall prove only one case, choose(2). Let A be N_α-open set in X_3 , thus A is N_α-open set in X_2, see (proposition(2.11)(1)),thus, $f^{-1}(A)$ is N_α-open (open) set in X_1resp. ,(since $f: X_1 \longrightarrow X_2$ is $N^*_{\acute{a}}$ ($N^{**}_{\acute{a}}$ - continuous)).

3.14. Theorem

If $f: X \longrightarrow Y$ is a mapping and $g: X \longrightarrow X \times Y$ is the graph mapping of f defined by $g(x) = (x, f(x))$ for every $x \in X$. Then

(1) If g is N_α-continuous, then f is N_α-continuous.

(2) If g is $N^*_{\acute{a}}$ -continuous, then f is $N^*_{\acute{a}}$ -continuous.

(3) If g is $N^{**}_{\acute{a}}$-continuous, then f is $N^{**}_{\acute{a}}$-continuous.

Proof ;We shall choose (2) and the proof of other statements by the same way. Let B be N_α-open set in Y, since X is N_α-open set in every topological space by (Remark (2.5)) then by (Theorem (2.7)) X×B is N_α-open set in X×Y, thus g^{-1}(X×B) is N_α-open set in X. But g^{-} (X×B) = f^{-1}(B). Thus f is $N^{*}_{\acute{a}}$-continuous.

3.15. Proposition

Let (X_1, τ_1), (X_2,τ_2) and $(X_3,\tau 3)$ be topological spaces and $f: X_1 \longrightarrow X_2$, $g: X_2 — X_3$ be mappings , then;

(1) If f is $N^{*}_{\acute{a}}$-continuous, g is N_α-continuous, then $g \circ f$ is N_α-continuous.

(2) If f is $N^{*}_{\acute{a}}$-continuous, g is $N^{*}_{\acute{a}}$-continuous, then $g \circ f$ is $N^{*}_{\acute{a}}$-continuous.

(3) If f is $N^{**}_{\acute{a}}$-continuous and g is $N^{*}_{\acute{a}}$-continuous, then $g \circ f$ is $N^{**}_{\acute{a}}$-continuous.

(4) If f is $N^{**}_{\acute{a}}$-continuous and g is N_α-continuous, then $g \circ f$ is continuous.

(5) If f is N_α-continuous and g is $N^{**}_{\acute{a}}$-continuous, then $g \circ f$ is $N^{*}_{\acute{a}}$-continuous.

(6) If f is N_α-continuous and g is continuous, then $g \circ f$ is N_α-continuous.

Proof; Obvious.

4. Contra N_α-Continuity

In this section, the concept of N_α-open set will be used to define new class of N_α-continuity called contra-N_α-continuous mapping. Some theorems will be proved.

4.1. Definition

Let $f: X_1 \longrightarrow X_2$ be a mapping, then f is called contra-N_α-continuous if for every an open set A in X_2, then $f^{-1}(A)$ is N_α-closed set in X_1.

4.2. Theorem

Let $f: X_1 \longrightarrow X_2$ be a mapping , The statements are equivalent:

(a) f is contra-N_α-continuous.

(b) $f^{-1}(A)$ is N_α-open set in X_1 , for every closed set A in X_2.

Proof: Obvious.

4.3. Theorem

Let $(X_1,\tau_1),(X_2,\tau_2)$ be topological spaces, and $f: X_1 \longrightarrow X_2$ be contra-N_α-continuous, then:

(i) $f|_{A1}$, $f|_{A2}$ are also, contra-N_α-continuous, such that $X_1=A_1 \cup A_2$, where A_1, A_2 are disjoint clopen sets in X_1.

(ii) $f|_{A:A} \longrightarrow X_2$ is also, contra-N_α-continuous, such that A is -N_α-open set in X_1.

(iii) f_A :$f^{-1}(A) \longrightarrow A$ is also, contra-N_α-continuous , where A is closed set in X_2 .

Proof: We shall choose (iii). Let B be closed set in A, since , A is closed in X_2 thus B is closed in X_2 ,since, $f: X_1 \longrightarrow X_2$ is contra-N_α-continuous then $f^{-1}(B)$ is N_α-open set X_1, since $f^{-1}(B) \subseteq f^{-1}(A) \subseteq X_1$ thus, by(proposition(2.11(1)), we get $f^{-1}(B)$ is N_α-open set in $f^{-1}(A)$.

The proof of others it follows by using proposition (2.11).

4.4. Theorem

Let $f: X_1 \longrightarrow X_2$, $g: X_2 \longrightarrow X_3$ be mappings. Then:

(1) If f is contra-N_α-continuous and g is continuous, then $g \circ f$ is contra-N_α-continuous.

(2) If f is $N^{*}_{\acute{a}}$-continuous and g is contra-N_α-continuous, then $g \circ f$ is contra-Ncontinuous.

Proof : Obvious.

4.5. Corollary

Let $f: A \longrightarrow \Pi X_\lambda$ be a contra N_α-continuous, where ΠX_λ is the family of topological spaces $\{ X_\lambda : {}_\lambda \in I \}$,then $f_\lambda : A \longrightarrow X_\lambda$ is also contra-N_α-continuous for each ${}_\lambda \in I$.

Proof: Let $f_\lambda = \rho_\lambda \circ f$, where ρ_λ is a projection mapping , also it is continuous for all ${}_\lambda \in I$,thus by (Th.(4.4)(1)) f_λ is contra -N_α-continuous, for each ${}_\lambda \in I$.

4.6. Theorem

Let $f : X \longrightarrow Y$ be a mapping and $g : X \longrightarrow X \times Y$ be the graph of f defined by $g(x) =(x, f(x))$,for every, $x \in X$. If g is contra-N_α-continuous, then f is contra-$N_\alpha$continuous.

Proof: It is similar to the proof of the Theorem (3.14) and hence omitted.

4.7. Theorem

Let $f : X \longrightarrow Y$, $g : X \longrightarrow Y$ be contra- N_α-continuous mappings, where Y is Ultra-T_2 space. Let $A = \{(a, b) : a ,b \in X$ such that $f(a) = g(b)\}$,then A is N_α-closed set .

Proof : We shall prove $\overset{c}{A}$ is N_α-open set, let (a,b) $\notin A$, thus $(a,b) \in \overset{c}{A}$, this means that $f(a) \neq g(b)$ in Y , since Y is Ultra-T_2- spaces ,thus there exist clopen sets G_1 ,G_2 such that $f(a) \in G_1$ and $g(b) \in G_2$ and $G_1 \cap G_2 = \emptyset$, since f , g are contra - N_α-continuous mappings, then $f^{-1}(G_1)$, $g^{-1}(G_2)$ are N_α-clopen sets ,hence by (Th.2.7) $f^{-1}(G_1) \times g^{-1}(G_2)$ is N_α-clopen set in $X \times X$,also $(a, b) \in f^{-1}(G_1) \times g^{-1}(G_2) \subseteq X \times X / A$, it follows A is N_α-closed set in $X \times X$.

Now, we shall give some applications about contra N_α-continuous mappings.

4.8. Theorem

Let $f: X_1 \longrightarrow X_2$ be a bijective contra- N_α-continuous mapping, where, X is locally indiscrete , $N_{\acute{a}}^{**}$ -regular space. Then the inverse image of T_2 -space under f is alsoT_2-space.

Proof : Let $x_1 \neq x_2$ in X_1 , since f is injective, then $f(x_1) \neq (x_2)$in X_2 , thus there exist G_1 , G_2 open sets contain $f(x_1)$, $f(x_2)$ in X_2 resp. , and $G_1 \cap G_2 = \emptyset$,thus $f^{-1}(G_1)$, $f^{-1}(G_2)$ are N_α-closed sets in X_1(since f is contra- N_α- continuous),since X_1 is $N_{\acute{a}}^{**}$ -regular space ,then $f^{-1}(G_1)$, $f^{-1}(G_2)$ are closed sets (see proposition (2.16)), since X_1 is locally indiscrete , then $f^{-1}(G_1)$, $f^{-1}(G_2)$ are open sets and contain x_1 , x_2 resp. ,also, $f^{-1}(G_1) \cap f^{-1}(G_2) = \phi = f^{-1}(G_1 \cap G_2)$ thus X_1 is T_2 –space.

4.9. Theorem

Let $f : X \longrightarrow Y$ be an open bijective, contra $N_\alpha$continuous ,where X is $N_{\acute{a}}^{**}$ - regular locally indiscrete space . If X is regular space , then Y is, also, regular-space .

Proof : Let $y \notin F$ where F is closed in Y since f is bijective , then there exists x such that $f(x)=y$,and $x= f^{-1}(y) \notin f^{-1}(F)$also, $f^{-1}(F)$ is N_α-open ,so it is an open(see proposition2.16 since X is locally indiscrete space ,then $f^{-1}(F)$ is closed ,since X is regular space , then there exist W_1 , W_2 open disjoint sets such that $x \in W_1$ and $f^{-1}(F) \subseteq W_2$, and $W_1 \cap W_2 = \phi$, thus $y=f(x) \in f(W_1)$, f $f^{-1}(F) = F \subseteq f(W_2)$, where $f(W_1)$, $f(W_2)$ are open sets (since f is an open mapping), also $f(W_1) \cap f(W_2) = f(W_1 \cap W_2)= f(\emptyset)=\emptyset$. Thus Y is regular space .

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